Adding and Subtracting Positive and Negative Numbers

Absolute Value

For any real number, the distance from zero on the number line is the *absolute value* of the number. The absolute value of any real number is represented with vertical bar symbols on each side of the number. The absolute value property is illustrated below.

**ABSOLUTE VALUE**

- The absolute value of a real number is equal to the distance from zero when the number is placed on the real number line. Absolute value is represented with vertical bars \(|\)\|.  

**Example:** The absolute value of \(-3\) = \(|-3| = 3\)

**Example**

Place 3 and \(-5\) on the real number line. Then find \(|3|\) and \(|-5|\).

Since \(-5\) is 5 units from zero, \(|-5| = 5\).

Since 3 is 3 units from zero, \(|3| = 3\).

**Example**

What is the absolute value of \(-3.6\) ?

\(-3.6\) is 3.6 units to the left of zero. Therefore, \(|-3.6| = 3.6\).

**Example**

Place \(-\frac{5}{2}\) and 3.2 on the real number line. Then determine which number has the larger absolute value.

\(-\frac{5}{2}\) is \(\frac{5}{2}\) or 2.5 units from zero. Therefore, \(\left|\frac{-5}{2}\right| = \frac{5}{2} = 2.5\).

Because 2.5 < 3.2, \(\left|\frac{-5}{2}\right| < |3.2|\) and the absolute value of 3.2 is larger.
Opposites
The opposite of a real number is defined as the real number that lies on the opposite side of zero on the real number line. This is summarized in the following table.

DEFINITION OF THE OPPOSITE OF A REAL NUMBER

- The opposite of a real number is the real number that lies on the opposite side of zero on the real number line and has the same absolute value.

Examples: The opposite of $-7.35$ is $7.35$.

The opposite of $5$ is $-5$.

- The opposite of a positive number is a negative number. The opposite of a negative number is a positive number.

Addition and Subtraction of Positive and Negative Integers

Integers consist of positive counting numbers, negative counting numbers, and zero. In order to add and subtract negative and positive integers, it is important to know basic addition and subtraction properties. Two critical addition and subtraction properties are given here.

**ADDITION AND SUBTRACTION PROPERTIES**

- Subtracting a real number is equivalent to adding its opposite.

  Example: $5 - (-2) = 5 + (\text{opposite of } -2) = 5 + 2$

- Adding a real number is equivalent to subtracting its opposite.

  Example: $4 + (-3) = 4 - (\text{opposite of } -3) = 4 - 3$

**Example** Use addition and subtraction properties to subtract $18 - (-3)$.

$$18 - (-3) = 18 + (\text{opposite of } -3)$$

$$= 18 + 3 = 21$$

**Example** Use subtraction and addition properties to add $12 + (-9)$.

$$12 + (-9) = 12 - (\text{opposite of } -9)$$

$$= 12 - 9 = 3$$
Example

Use subtraction and addition properties to subtract $15 - (-15)$.

$15 - (-15) = 15 + (\text{opposite of } -15)$

$= 15 + 15 = 30$

When adding positive and negative integers, there are different strategies which one may use. Three strategies are summarized here.

**STRATEGIES FOR ADDING POSITIVE AND NEGATIVE INTEGERS**

**STRATEGY 1 - NUMBER LINE METHOD**

- Use a number line and locate the point corresponding to the first number.

  If the second number is positive, move to the right on the number line by the number of units equal to the second number.

  If the second number is negative, move to the left on the number line by the number of units equal to the second number.

Example: Add $-1 + (-7)$.

Locate $-1$ on the number line. Then move 7 units to the left.

The result of moving 7 units to the left of $-1$ is $-8$.

Therefore, $-1 + (-7) = -8$. 

**STRATEGIES FOR ADDING POSITIVE AND NEGATIVE INTEGERS**
STRATEGY 2 - ABSOLUTE VALUE METHOD

- To add a negative number to a positive number, find the difference of the numbers absolute values. The answer consists of this difference with the sign of the number with the larger absolute value.

Example: Add $-17 + 6$.

The absolute values are $|-17| = 17$ and $|6| = 6$.

The difference of the absolute values is $17 - 6 = 11$.

Since $|-17| > |6|$, a negative sign is used in the answer.

Therefore, $-17 + 6 = -11$.

- To add two negative numbers together, add the absolute values of the numbers. The answer is the negative of this sum.

Example: Add $-18 + (-34)$.

The absolute values are $|-18| = 18$ and $|-34| = 34$.

The sum of the absolute values is $18 + 34 = 52$.

The answer is the negative of 52.

Therefore, $-18 + (-34) = -52$.

Example: Add $-17 + 23$.

The absolute values are $|-17| = 17$ and $|23| = 23$.

The difference of the absolute values is $23 - 17 = 6$.

Since $|23| > |-17|$, the answer is positive 6.
STRATEGY 3 - PAYCHECK-DEBT ANALOGY

- Think of positive numbers as “paychecks” and think of negative numbers as “debts”.

Example: Add \((-15) + (-23)\).

\[-15\] may be thought of as a $15 debt. \[-23\] may be thought of as a $23 debt. When two debts are added together, a bigger debt is always the result.

$15 debt + $23 debt = $38 debt

Since a debt represents a negative number, this $38 debt represents \(-38\).

Therefore, \((-15) + (-23) = -38\).

Example: Add \(-50 + 34\).

\[-50\] may be thought of as a $50 debt. 34 may be thought of as a $34 paycheck. When a $34 paycheck is added to a $50 debt, the result will be a debt that is equal to the difference of the two amounts.

$50 debt + $34 paycheck = $16 debt

Since a debt represents a negative number, this $16 debt represents \(-16\).

Therefore, \(-50 + 34 = -16\).

Example: Add \(-14 + 25\).

This may be thought of as adding a $14 debt to a $25 paycheck. Since the “paycheck” is bigger, the result is a positive amount of $11.

\(-14 + 25 = 11\)
Subtraction of Positive and Negative Integers

The simplest way to successfully subtract positive and negative integers is to rewrite each subtraction problem as an addition problem. Then, use the rules for addition.

PROCEDURE FOR SUBTRACTING POSITIVE AND NEGATIVE INTEGERS

1. Rewrite the subtraction problem as an addition problem. Subtraction of a number is equivalent to adding the opposite of the number.
2. Use methods for addition of integers.

Example Subtract $-15 - 34$.
Rewrite this as $-15 + (\text{opposite of } 34) = -15 + (-34)$.
$-15 + (-34) = -49$

Example Subtract $-45 - (-23)$.
Rewrite this as $-45 + (\text{opposite of } -23) = -45 + 23$.
$-45 + 23 = -22$.

Example Subtract $-57 - (-21)$.
Rewrite this as $-57 + (\text{opposite of } -21) = -57 + 21$.
$-57 + 21 = -36$.

Rewrite this as $15 + (\text{opposite of } 24) = 15 + (-24)$.
$15 + (-24) = -9$. 